Optimal Frequency of Portfolio Evaluation in a Choice Experiment with Ambiguity and Loss Aversion*

Charles Bellemare[1], Sabine Kröger[1], and Kouamé Marius Sossou[1]

[1]Economics Department, Laval University
[1]email: *charles.bellemare@ecn.ulaval.ca, sabine.kroger@ecn.ulaval.ca, kouame-marius.sossou.1@ulaval.ca*

Abstract

We estimate a structural model using data from a novel experiment to investigate how optimal frequency of portfolio evaluations balances the opposing effects of ambiguity and loss aversion. Investors in the experiment face initial ambiguity concerning return distributions for an asset. They observe draws from the true return distribution of the asset, allowing them to reduce their ambiguity through time. We exploit portfolio choices and stated beliefs over possible return distributions to estimate preferences and ambiguity updating rules. We find that 70% of investors benefit from high frequency of portfolio evaluation, reflecting the dominating effect of ambiguity aversion over loss aversion.

JEL codes: C25, C91, D81, G11
Keywords: portfolio choice, feedback frequency, narrow bracketing, ambiguity aversion, loss aversion.

*The authors thank the audience at the CESifo Workshop On Subjective Expectations and Probabilities in Economics and Peter Wakker for very helpful comments and suggestions.

1 Introduction

Ambiguity in financial markets is a natural consequence of investors being unaware of the objective probabilities of asset returns. Recent models of ambiguity suggest that ambiguity-averse investors dislike not knowing objective probabilities, pushing them towards more conservative investments with known returns. While preferences over ambiguity may be stable over time, ambiguity itself is dynamic rather than static. Increasing portfolio evaluation frequency enables learning, which can diminish ambiguity and leads to more risk taking.

However, the literature in behavioral finance suggests the opposite effect, namely that increasing the portfolio evaluation frequency may instead backfire and cause more conservative investments. Benartzi and Thaler (1995) argue that conservativeness in financial investments may reflect loss aversion in combination with myopia, also referred to as narrow bracketing. Myopic loss-averse investors evaluate their portfolio too frequently. As a result, they mentally cumulate short-term losses which are amplified because of investors' loss aversion, considerably reducing the attractiveness of risky investments. Experimental support for this hypothesis in the context of risk (with known probabilities) includes Gneezy and Potters (1997) and Bellemare, Krause, Kröger, and Zhang (2005). In contrast to ambiguity-averse investors, myopic loss-averse investors benefit from less frequent portfolio evaluation.

The evidence discussed above suggests that ambiguity aversion and myopic loss aversion can co-exist in the presence of ambiguity. Addressing the normative question of the optimal frequency of portfolio evaluation in the presence of ambiguity requires joint measurement of preferences concerning ambiguity and losses as well as how investors update their beliefs about the possible distributions of asset returns. Measuring the latter is essential to the analysis, since the speed at which investors reduce their ambiguity will affect whether or not ambiguity aversion can potentially dampen the effects of loss aversion over a long investment horizon. Barberis and Thaler (2003) conjecture that loss aversion will dominate ambiguity aversion as the most relevant preference to explain conservative investments, suggesting that long investment horizons would be optimal in general. We are not aware of existing empirical work supporting this conjecture.

This study analyzes data from a novel experimental design in which investors are asked to make repeated portfolio choices facing initial ambiguity concerning the distribution of returns of one of the available assets. Ambiguity is structured in the spirit of Klibanoff, Marinacci, and Mukerji (2005): investors are presented two possible returns distributions for the ambiguous asset, one of which has been randomly chosen to determine the asset returns. At exogenously-varying time intervals, investors observe returns drawn from the true distribution. Additionally, they are asked to express their ambiguity by stating and updating probabilistic beliefs about which of the two distributions is the true distribution determining the asset's returns. Our analysis exploits exogenous variations in investment horizons, updated probabilistic beliefs, and portfolio choices to identify and estimate the joint distribution of key behavioral components including investors' ambiguity aversion, loss aversion, risk aversion (gain and loss domains) as well as their ambiguity updating rule. We model this joint distribution flexibly using a finite mixture approach placing limited shape restrictions.

Our design and analysis significantly extends previous work on the determinants of portfolio choices under ambiguity. Charness and Gneezy (2010) conduct a between-subject classroom experiment in which subjects are assigned to treatments with various levels of risk and ambiguity. They exclude learning by design with each subject making a single investment decision in their ambiguity treatments. What is more, the between-subject nature of the experiment does not inform on the joint distribution of loss aversion and ambiguity aversion. In contrast, our experimental design allows to observe the same investor making decisions both in ambiguous and risky environments. Iturbe-Ormaetxe, Ponti, and Tomás (2016) extend the between-subjects risk experiment of Gneezy and Potters (1997) by adding a treatment in which subjects do not know the probability distributions of the lotteries played. They replicate the results of Gneezy and Potters (1997) under both risk and ambiguity, suggesting loss aversion dominates ambiguity aversion in their sample. They do not recover distributions of preferences nor do they derive the optimal frequency of portfolio evaluation.

Our main results can be summarized as follows. First, we find that investors are slow to update their probabilistic beliefs regarding the true distribution relative to predictions based on Bayes rule. Our estimates suggest this occurs because investors place relatively little weight on new information concerning the true distribution relative to their prior. As a result, initial ambiguity persists for a large share of investors throughout the experiment. Second, the estimated finite mixture model captures considerable heterogeneity in preferences across seven different preference types. While average preference estimates are in line with corresponding values found in the literature, our analysis identifies 7 different preference types. Broadly speaking, 58% of investors are estimated to be simultaneously driven by ambiguity and loss aversion. We use our estimates to predict the distribution of the optimal frequency of portfolio evaluation in our sample population. Our predictions suggest that approximatively 70% of investors prefer the highest evaluation period frequency possible. This prediction reflects the dominating effects of ambiguity aversion and (slow) belief updating, and runs counter to the conjecture that loss aversion dominates ambiguity aversion in portfolio choices (see Barberis and Thaler (2003)). The remainder of the paper is structured as follows. Section 2 describes the experiment. Section 3 presents the econometric model. Section 4 presents the main results of the paper. Section 5 concludes.

2 Experiment

Participants in the experiment were placed in the role of investors who make investment decisions over 30 periods in each of three treatments. These 30 periods were broken down in subsequent investment intervals capturing investment horizons of one, three, or five periods. Investors were told at the beginning of each investment interval to allocate a period-specific endowment between a stock and a risk-free asset. To simplify the experiment, the endowment for each period of a given investment interval was the same, set to 100 experimental currency units. Investors had to decide on the share of the endowment to allocate to each asset for each period of the investment interval.

Returns of the risk-free asset (in %) were drawn uniformly from the [1,5] interval for each investor at the beginning of each investment interval. Returns of the risk-free asset were thus kept constant within but not across investment intervals. The stock generated returns at the end of every period. Investors were presented two possible returns distributions for the stock at the beginning of the experiment. Both distributions involved two possible returns on the amount invested in the stock at the end of each period, either a loss of 10% or a gain of 8%. Probabilities of experiencing a loss or gain in a given period varied across both distributions. Figure 1 presents a screenshot of the main decision screen in the experiment. Histograms of both possible distributions were presented on the left hand side of the screen. Positioning of both histograms (top/bottom) was randomized across investors. Information about the return of the risk-free asset was presented immediately below these histograms. The period-specific endowment and duration of the investment interval was presented below. Investors could enter their allocation (a number between 0 and 100) in the corresponding entry boxes. The computer verified that the amounts entered totalled the available endowment and prompted investors to correct (if needed) their allocation.

Investors did not initially know which of the two possible distributions was the true distribution actually generating stock returns and thus faced initial ambiguity concerning the returns of the stock. The right-hand side of the screen displayed the distribution of all past random draws taken from the true distribution generating stock returns at the beginning of each period. This space was empty at the start of each treatment. Figure 1 presents an example of this histogram after 15 periods. As we can see from this example, the distribution of past draws after 15 periods begins to resemble the top histogram of the left hand side of this screen. Each investor was asked to state their probabilistic beliefs concerning the likelihood that each possible distribution was the true distribution for the stock. Investors entered this information (chances out of 100) in the entry boxes next to each possible distribution. The computer program warned investors when beliefs did not sum to 100. Investors could thus learn through time the true distribution by keeping track of all past draws and updating their probabilistic beliefs when required. A key element of this design is that investments decisions in early periods were made under ambiguity (with uncertainty about the true distribution) while decisions in later periods were made closer to an environment under risk. The later occurs when investors state knowing with probability of 1 the true distribution generating stock returns.

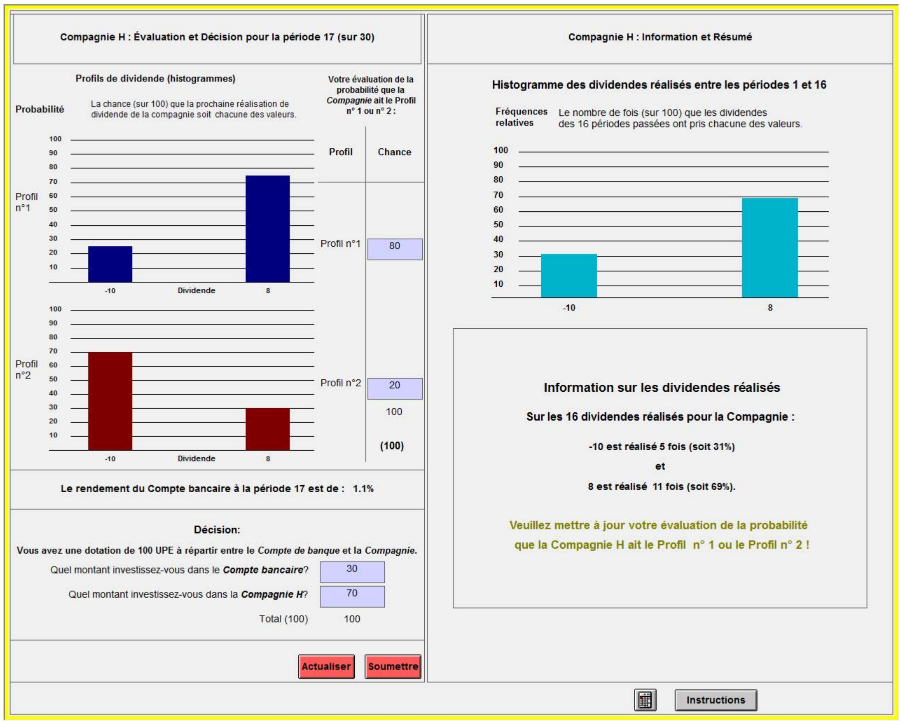

Figure 1: Screenshot of the decision screen.

Following Bellemare et al. (2005), the experiment implemented three treatments (labelled H, M, and L) varying the investment horizon and the frequency of feedback about portfolio returns. The investment interval in treatment H is a single period and investors received feedback at the end of each investment interval. This treatment thus combines high frequency of portfolio evaluation with high frequency of information feedback. Treatment L imposes an investment interval of either 3 or 5 periods. Investors in this treatment had to allocate the period-specific endowment for al periods of the investment interval. They also received feedback at the end of the investment interval. This treatment thus combines low frequency of portfolio evaluations with low feedback frequency. Importantly, investors in treatment L observe the aggregate return on the stock over the interval, not the returns of each period in the interval. While it was possible for investors to back out the number of gains and losses in the interval that are consistent with the posted aggregate return, it was not always possible to back out the exact sequence of draws in the interval. For example, an investor may infer that he/she suffered two loses and one gain over a three period investment interval, but does know the ordering of these events. The updating rule presented in the next section assumes investors in treatment L consider all sequences consistent with the observed aggregate return when updating their uncertainty about the true return distribution.

Treatment M is a mixture of treatments H and L. Investor in treatment M had to allocate the period-specific endowment for all periods of the investment interval (3 or 5 periods) (as in treatment L) while receiving feedback about their portfolio after every period on the investment interval (as in treatment H). Treatment M allows to assess whether any behavior compatible with loss aversion is driven by the frequency of investment decisions or the frequency of portfolio evaluations (see Bellemare et al. (2005) for further discussion and results). Figure 7 in the Appendix presents the feedback screens for each treatment of the experiment. Investors in treatments H and L observed at the end of each investment interval their returns on investing in the stock and their profits from investing in the risk-free asset. Investors in treatment M could assess this information in every period of the investment interval.

Despite the fact that the experiment was computerized, investors were asked to record on a sheet of paper their beliefs, decisions, and realized returns after each investment interval. This was done to raise awareness of investors about their losses and gains during the experiment. Figure 8 in the Appendix presents the record sheets used for all treatments.[1]

We conducted a total of 15 sessions in 2016 and 2017 at the Laboratory of Experimental Economics at Universit Laval in Québec, Canada. We recruited participants from a database containing approximately 500 persons who had signed up to participate in

[1]Related papers also requiring investors to record outcomes on a sheet of paper include Gneezy and Potters (1997) and Bellemare et al. (2005).

economic experiments. A total of 113 persons participated in our experiment. Amongst them, 57 faced investment intervals of three periods, while 56 faced investment intervals of five periods. Participants were also randomly assigned to one of the three treatment orders, HML (N=39), LMH (N=37) and MLH (N=37). Investors began the experiment by watching video instructions presenting them computer screens of the experiment and the related tasks. The experiment was computerized (z-Tree, Fischbacher (2007)). During the experiment, investors could access online instructions and a calculator by clicking on the corresponding on-screen buttons. At the end of the experiment, investors completed a questionnaire including standard demographic questions. The experiment lasted on average 90 minutes. Investors received the total profit they made during the experiment and an additional \$ 5 show-up fee. The average experimental compensation was \$ 25.3 CAD with minimal earnings of \$ 15 and maximal earnings of \$ 33.75 CAD.

25 of the 113 investors completed the experiment with very erratic and noisy ambiguity updating behavior which cannot be reconciled with the empirical model described in the next section. In what follows we discard these investors and perform our analysis with the remaining 88 investors.[2]

Figure 2 plots the distribution of experimental periods starting from which investors ($N = 88$) correctly assign a subjective probability greater than 90% on the true return distribution of the stock in treatment H (left panel). Right panel plots the corresponding distribution assuming investors update their ambiguity using Bayes rule. The last right hand vertical bar of each panel captures investors who had not placed 90% probability on the true distribution after 30 periods. We find that close to 45% of investors fail to assign a probability greater than 90% even after observing 30 draws (one for each period). By comparison, this share is slightly below 15% if investors where perfect Bayesian updaters. Moreover, we find a clear shift to the left of the observed distribution relative to the Bayesian prediction. Overall, investors update their ambiguity slowly relative to Bayes

[2]Table 5 in the Appendix presents frequencies of investors dropped per treatment sequence and investment interval lengths (3 or 5 periods in treatments L and M). There is no clear association between these variables and investors dropped from the analysis.

rule.

Figure 3 reproduces the left panel of Figure 2 for treatments M and L as a function of the number of periods of the investment interval. As for treatment H, we find that close to 45% of investors fail to assign a probability greater than 90% to the true distribution at the end of treatment M. Distributions over 3 and 5 periods in the later treatment tend to agree with that observed in treatment H. Observed distributions in treatment L are more concentrated to the right relative to treatment M, suggesting investors update their ambiguity more slowly when presented aggregate returns over the investment interval rather than period-specific returns. Consistent with treatment H, predictions using Bayes rule (Figure 4) suggest investors will identify the true distribution more quickly in both treatments.

Figure 4 replicates Figure 3 assuming all investors are Bayesian updaters. We find that treatment L slows down learning relative to the other treatments even under Bayesian updating. The first graph column of Figure 4 compares both treatments for investment intervals of 3 periods. We find that Bayesian updaters would begin assigning a probability of 90% or more starting in period 6 in treatment L, but would do so already after 3 periods in treatment M. The second graph column presents the corresponding predictions for investment intervals of 5 periods. Here, we find that Bayesian updaters would begin assigning a probability of 90% or more starting in period 20 in treatment L, but would do so already after 5 periods in treatment M. As we formalize in the next section, learning in treatment L is impeded because investors only observe the aggregate return over the investment interval. While they can back out the exact number of gains and losses in the interval from this aggregate, they are not aware of the particular sequence of gains and losses that generated the aggregate return. This differs from treatments H and M where all sequences of draws are observed over the investment intervals. Intuitively, taking into account the multiplicity of possible sequences or losses and gains consistent with an given aggregate return limits learning under treatment L.

3 Econometric model

In this section, we present the behavioral model we estimate using the data collected. The model specifies the joint distribution of risk (loss and gain domains), ambiguity, and loss aversion preferences, along with belief updating rules governing how investors update their ambiguity. To simplify notation, we present the problem for a given investor and omit investor subscripts in this section.

Our design requires an investor to allocate at the beginning of each investment interval an endowment W between an ambiguous asset (the stock) and a safe asset. Let $\mathcal{T}_i$ denote the set of elicitation periods where investors completed the uncertainty and investment decisions. The size of this set varies across treatments in our experiment – 30 investment intervals in treatment H, and either 6 or 10 investment intervals in treatments M and L, depending on the duration of these intervals (3 or 5 periods).

The ambiguous asset comprises two potential returns distributions and one has been determined to generate the asset's returns during the corresponding part of the experiment. We model the choice problem of investors in the framework of Klibanoff et al. (2005). Investors decide at the beginning of each investment interval the amount X_t with ($0 \leq X_t \leq W$) to allocate to the ambiguous asset in order to maximize the following objective function :

$$V(X_t) = \phi^{-1}\left(\mu_t^1 \phi\left(EU^1(X_t)\right) + \mu_t^2 \phi\left(EU^2(X_t)\right)\right) \tag{1}$$

where μ_t^j denotes the subjective probability assigned by the investor at period t to distribution j being the true distribution (where $\mu_t^1 + \mu_t^2 = 1$ at all periods). Ambiguity is present as long as $\mu_t^1 \mu_t^2 \neq 0$ – ambiguity resolution requires that an investor places a probability of 1 on one of the two possible return distributions being the true one. $EU^j(X)$ denote the expected utilities (defined below) of allocating X_t to the ambiguous asset conditional on distribution j being the true one generating returns.

$V_t(X)$ can be interpreted as the certainty equivalent of not knowing the expected utility to maximize at time t. As the experiment does not allow accumulation of savings, the choice problem changes across periods primarily because investors can update μ_t^j about the which if the two possible distributions is the true one.

The function $\phi(.)$ describes the investors attitude towards ambiguity and is given by:

$$\phi(x) = \begin{cases} \frac{1-\exp(-\alpha x)}{\alpha} & \text{if } \alpha \neq 0; \\ x & \text{if } \alpha = 0; \end{cases} \tag{2}$$

where α is the ambiguity aversion parameter. Negative, null, and positive values of α respectively capture ambiguity seeking, neutral, and averse investors. Note that $\phi(\cdot)$ (and hence α) plays no role in determining X_t when ambiguity is resolved and the investor has assigned a probability 1 to one of the two possible return distributions. In the later case, investors maximize $EU^j(X_t)$ for $\mu_t^j = 1$.

Given a distribution j, we model the investment choice using prospect theory by specifying :

$$EU^j(X_t) = \sum_{k=0}^{h} p_k^h U\left(W_k^h(X_t)\right) \tag{3}$$

where we use the following piecewise power value function :

$$U(a) = \begin{cases} (a - \overline{w})^{\theta} & \text{if } a \geq \overline{w}; \\ -\lambda(\overline{w} - a)^{\theta'} & \text{if } a < \overline{w}; \end{cases} \tag{4}$$

where $\theta > 0$ and $\theta' > 0$ are the risk aversion coefficients for gains and losses respectively, $\lambda > 0$ denotes the loss aversion parameter, and $\overline{w}$ captures the reference point below and above which investors experience a psychological loss or gain (see e.g. (Kahneman and Tversky, 1979)). Values of θ less than 1 imply concavity of $U(a)$ and thus risk aversion in the gain domain. Conversely, $\theta' < 1$ implies convexity of $U(a)$, consistent with risk-seeking in the loss domain. Numerous previous studies support these values (see Wakker (2010) for an overview). Our analysis does not impose sign restrictions on these parameters, such that investors may for example be risk-averse in the loss domain as well (when $\theta' > 1$). Recent studies have found evidence that a majority of individuals are loss averse ($\lambda > 1$), with a small minority of individuals exhibiting gain seeking behavior ($\lambda < 1$) (see Abdellaoui, Bleichrodt, and Paraschiv (2007)). Our analysis will consequently place no restrictions on this parameter as well. The risk-free payoff is often used has reference point in the portfolio choice problem (see for instance Barberis and Huang (2009); Bernard and Ghossoub (2010); He and Zhou (2011) and De Giorgi and Legg (2012)). Following this literature, we define $\overline{w}$ as the outcome when investing the totality of W in the risk-free asset for the duration of the investment interval given risk-free return r_{ft}. In practice, we set $\overline{w} = (1 + r_{ft})W$ in treatment H, and $\overline{w} = (1 + hr_{ft})W$ in treatments M or L where $h \in \{3, 5\}$ is the investment interval duration, which varies exogenously across investors.

The number of terms in the summation (3) depends on the duration of the investment interval which varies across treatments. Investments are made every round ($h = 1$) under treatment H. The return of the ambiguous asset under each possible distribution j are given by a simple mixed binary lottery $L^j : (p_j : -0.1, 1-p_j : 0.08)$, where p_j varies across j (see Section 2) and $(-0.1, 0.08)$ are the possible returns from investing in the ambiguous asset. Hence, it follows from that

$$
\begin{aligned}
EU^j(X_t) &= p_j U\left((1-0.1)X_t + (1+r_{ft})(W - X_t)\right) && (5)\\
&+ (1-p_j)U\left((1+0.08)X_t + (1+r_{ft})(W - X_t)\right) && (6)
\end{aligned}
$$

The summation is more complicated as the investment interval duration increases. In the case of an investment interval covering 3 periods, investing X_t in each period can yield either three independent losses of 10%, three gains of 8%, two losses and one gain, and one loss and two gains. The probabilities p_k^h for $k = 0, 1, 2, 3$ for $h = 3$ capture the induced lottery over these 4 potential outcomes $W_k^h(X_t)$. Similarly, there are 6 possible payoff outcomes for investment intervals associated with 5 periods.[3]

[3]Let $L : (p : a, 1-p : b)$ be a mixed binary lottery. We have that h i.i.d draws of L induce a compound lottery of length $h+1$ (the number of distinct outcomes in the lottery) defined as follows:

$$
\begin{aligned}
L_h &= \left(C_h^k \, p^k(1-p)^{h-k} : ka + (h-k)b \right)_{k=0}^{h} && (7)\\
&= \left((1-p)^h : hb, hp(1-p)^{h-1} : a + (h-1)b, \cdots, C_h^k p^k(1-p)^{h-k} : ka + (h-k)b, \cdots, hp^{h-1}(1-p) : (h-1)a + b, p^h : ha \right)
\end{aligned}
$$

Denote by $r_k^h = ka + (h-k)b$ and $p_k^h = C_h^k \, p^k(1-p)^{h-k}$, $(0 \leq k \leq h)$, the compound returns and their realization probabilities of the compound lottery L_h respectively. Moreover, C_h^k is an integer capturing the number of draw sequences consistent with the same compound return. At the end of an investment

Model estimation

Investors faced 101 choice alternatives in each investment interval ($X_t \in \{0, 1, ..., 100\}$ for $t \in \mathcal{T}_i$). Let ($V_{at}(\boldsymbol{\beta})$) denote the certainty equivalent associated with alternative a in the set of possible investment choices, where $\boldsymbol{\beta}$ denotes the vector of parameters entering (1). The model presented above does not allow for noise in decision making and possible heterogeneity in preferences. We introduce noise in decision making by adding an additive noise term to the deterministic part of ($V_{at}(\boldsymbol{\beta})$)

$$\overline{V}_{at} = V_{at}(\boldsymbol{\beta}) + \tau\epsilon_{at} \tag{10}$$

where ϵ_{at} are i.i.d. draws from the Type 1 extreme value distribution and τ is the noise parameter. We assume investors choose alternative n that maximizes contaminated certainty equivalent $\overline{V}_{at}$.

In order to take into account individual heterogeneity, we assume that $\boldsymbol{\beta}$ and τ vary across investors given a joint distribution F. We model this heterogeneity using a discrete finite mixture approach. In particular, we assume the population of investors comprises C classes, or types. Let $\{(\boldsymbol{\beta}_c, \tau_c) : c = 1, \cdots, C\}$ be the support of the joint distribution and s_c denote the share of investors in class c.

The probability that an investor in a given class c chooses alternative n is

$$L_{nt}(\boldsymbol{\beta}_c, \tau_c) = \frac{\exp(V_{nt}/\tau_c)}{\sum_{a=1}^{101} \exp(V_{at}/\tau_c)} \tag{11}$$

interval of h periods, the final outcome resulting of allocation of amount X_t in L and $W - X_t$ in the risk-free asset if the compound return r_k^h realized is:

$$W_k^h(X_t) = (1 + r_k^h)X_t + (1 + hr_f)(W_0 - X_t) \tag{8}$$

Therefore, an investor allocating X_t to the ambiguous asset and $W - X_t$ to the risk-free asset for an investment interval of h periods faces the following prospect:

$$\mathcal{X} = \left(p_k^h : W_k^h(X) \right)_{k=0}^{h} \tag{9}$$

Each investor made a sequence of $\mathcal{T}_i$ investment choices. The likelihood contribution of a given choice sequence for an investor in class c is given by

$$K_c(\boldsymbol{\beta}_c, \tau_c) = \prod_{t=1}^{\mathcal{T}_i} L_{nt}(\boldsymbol{\beta}_c, \tau_c) \tag{12}$$

The likelihood contribution of an investor is computed by averaging (12) over the C classes, yielding

$$P(\Theta) = \sum_{c=1}^{C} s_c K_c(\boldsymbol{\beta}_c, \tau_c), \quad \text{with } \Theta = \{(\boldsymbol{\beta}_c, \tau_c, s_c),\, c = 1, \cdots, C\} \tag{13}$$

The sample log likelihood is obtained by summing the natural logarithm of (13) over all investors in our experiment. Following Bhat (1997) and Train (2008), we estimate the model parameters using the iterative Expectation-Maximization (EM) algorithm. The estimation algorithm is applied for a given number of classes C. In practice, the number of classes C is unknown a priori. Information criterions such as Akaike (AIC) and Bayes (BIC) are typically used to identify the most relevant value for C for a given model and data.[4] We compute both criterions for a sequence of models estimated for using different number of classes $C = 1, 2, 3, \ldots$. Our preferred model corresponds to the one associated with the smallest AIC and/or BIC.

Modelling distribution uncertainty updating

Investors state their uncertainty about the true return distribution at all 30 periods in treatment H, but only at each 3 or 5 periods in treatments M and L. Let $\mu_t^j, j = 1, 2$ denote the subjects expressed uncertainty over the two possible distributions j generating the stock returns at period t during the experiment, with $\sum_{j=1}^{2} \mu_t^j = 1$. Furthermore, let $\pi_{t-\iota}^j$ denote the probability of the most recently observed return given distribution j. This probability is either p_j for a loss and $1 - p_j$ for a gain in treatments H and M. Investors in

[4] AIC is calculated as $-2L_C + 2R_C$, where L_C is the log likelihood function value at the optimal solution for C classes, and R_C is the number of parameters estimated with C latent classes. BIC is similarly defined but considers sample size in addition to the number of parameters: $-2L_C + R_C \ln N$, where N corresponds to the number of investors in our experiment.

treatment L observe the aggregate return over the investment interval (3 or 5 periods), not the sequence of draws that generated this aggregate. It follows that $\pi^j_{t-\iota}$ the probability of the observed return over the investment interval is equal to the corresponding value of p^h_k (see equation (3)). Note that the later takes into account that investors do not know which sequence of returns generated the observed aggregate returns at the end of the investment interval.

Following Bellemare, Kröger, and Sossou (2018), we assume that subjects update their uncertainty μ^j_t given the observed returns from the true distribution using the following modified Bayes rule

$$\mu^j_t = \frac{\left(\mu^j_{t-\iota}\right)^{\delta_1}\left(\pi^j_{t-\iota}\right)^{\delta_2}}{\sum_{j=1}^{2}\left(\mu^j_{t-\iota}\right)^{\delta_1}\left(\pi^j_{t-\iota}\right)^{\delta_2}} \quad \text{for } j = 1, 2 \tag{14}$$

where ι captures the number of periods between feedback on realized draws (1, 3, or 5 according to treatment). The parameters (δ_1, δ_2) control the weight placed by investors on their prior uncertainty $\mu^j_{t-\iota}$ and on the most recent information $\pi^j_{t-\iota}$. Investors relying predominantly on the most recent return would place a relatively lower weight on their prior ($\delta_2 > \delta_1$), leading to behavior originally referred to as base rate neglect. Conversely, ($\delta_2 < \delta_1$) would imply insensitivity to recent information and a reluctance to update the prior. We pool data from the three treatments and estimate (δ_1, δ_2) for each investor using non-linear least squares by solving:

$$\min_{(\delta_1,\delta_2)} \sum_{t\in\mathcal{T}_i}\sum_{j=1}^{2}\left(\mu^j_t - \frac{\left(\mu^j_{t-\iota}\right)^{\delta_1}\left(\pi^j_{t-\iota}\right)^{\delta_2}}{\sum\limits_{j=1}^{2}\left(\mu^j_{t-\iota}\right)^{\delta_1}\left(\pi^j_{t-\iota}\right)^{\delta_2}}\right)^2 \tag{15}$$

Notice that $\mu^j_{t-\iota}$ is observed for all elicitation periods in treatments H and L. For treatment M, investors observe each draw in succession before adjusting their investment strategy and updating their uncertainty at the following elicitation period. We assume investors in treatment M continuously update their prior after observing each draw ($\iota = 1$), akin to treatment H. This assumption implies that $\mu^j_{t-\iota}$ are not observed between elicitation periods in treatment M. We address this issue using (14) to solve recursively for missing values of $\mu^j_{t-\iota}$ given (δ_1, δ_2) and observed draws from the true distribution. This recursion is solved every time (δ_1, δ_2) are changed when solving (15).

Identification and calibration of experimental design

Variations of the duration of the investment intervals and the frequency of information feedback have been used previously to test for the existence of myopic loss aversion. We use these variations in combination with variations in returns for the risk-free asset and varying levels of ambiguity for the stock return to identify the mixture of risk, ambiguity, and loss aversion preferences in our population. More precisely, risk and loss aversion are the only preference parameters determining investments choices once an investor sets $\mu^j_t = 1$ for one of the possible distributions. For investment periods with μ^j_t at or close to 1 for some j, identification of risk and loss aversion comes from exogenous variations of both the risk-free rate r_{ft} and duration of the investment period h which affects the probabilities of experiencing a loss or a gain as well as the reference point $\overline{w}$. Given identification of $(\theta, \theta', \lambda)$, investment decisions in periods where μ^j_t differs from 1 for some j identify ambiguity preferences captured by α. More generally, variations of μ^j_t vary the importance of ambiguity aversion relative to other preferences and thus serves to separate these elements. Finally, variations of stated probabilistic beliefs throughout the experiment identifies the updating rule used by investors given the draws observed from the true distribution determining the stock returns.

Our experimental design involves many elements including the distribution of risk-free asset returns, possible return distributions for the stock, treatments, investment interval durations, etc.. We conducted an extensive Monte Carlo analysis prior to running our experiments varying these design elements in order to select those most useful to identify preferences and updating rules. We conducted this analysis by drawing samples of investors and decisions assuming preferences (ambiguity, loss, and risk aversion) were jointly normally distributed with realistic levels of correlation and heterogeneity. The updating rules were specified as above. Design elements that were kept were those maximizing the sampling probability of correctly detecting the assumed mixture of preferences and updating rules. The assumption that preferences are jointly normally distributed is rejected in our data which led to the specification of the finite mixture model presented above.

4 Results

The finite mixture model presented in Section 3 was estimated for C classes varying from one to seven[5]. Table 4 in the Appendix presents estimated information criterias of these models. The log-likelihood values reveal considerable improvement in the model fit (from -17519 with 1 class to -16014 with 7 classes) as classes are progressively added. Moreover, both AIC and BIC are their lowest with 7 classes. As a result, our preferred model sets $C = 7$. Table 5 in the Appendix further presents the predicted posterior probabilities that each investor belongs to each of the 7 estimated classes. From Bayes rule the posterior probability that an investor belongs to class c given their choices is

$$h_{nc}(\Theta) = s_c K_c(\boldsymbol{\beta}_c, \tau_c)/P(\Theta) \tag{16}$$

We find that 96.6% and 92% of investors can be assigned to one of the classes with posterior probability in excess of 0.9 and 0.95 respectively. Given their choices, the model is thus able to assign almost all investors to one of the classes with very high probability.

Estimation results are shown in Table 1. We find that estimated coefficients are in general precisely estimated. We discuss first the average parameter values across the seven classes reported at the bottom of the table. We find close to linear utility in the gain domain ($\hat{\theta} = 0.941$) reflecting risk neutrality, but risk-seeking in the loss domain with an average value of $\theta' < 1$. Moreover, average loss aversion is measured at 2.158. All three averages are consistent with numerous value function estimates reported in the prospect theory literature (see Barberis (2013) for a recent survey). The average estimated ambiguity aversion parameter value is 0.23, on the side of ambiguity aversion. We discuss below the magnitude of the later value.

[5]We experienced significant numerical instability for some classes when C exceeded 7.

Analysis of class specific preference estimates suggests these average values mask considerable heterogeneity in the population. In particular, we distinguish two noisy classes (classes 2 and 3 representing 19.3% of our sample), where subjects in these classes have relatively high noise parameters compared to those of other classes. Further, classes 5 and 6 are the two largest and represent 50% of the population. These classes are composed of subjects, who are risk-averse over gains, risk-seeking over losses and ambiguity averse. Subjects in class 5 (23.7%) on the other hand are loss-seeking, while those in class 6 (26.3%) are loss averse. Moreover, subjects in classes 3 and 4 (18.2%) are loss-averse and risk-seeking over both gains and losses. Subjects of these two classes differ in their attitude towards ambiguity. In fact, if subjects in class 3 (6.8%) are ambiguity seeking, those in class 4 (11.4%) are ambiguity averse. Risk-seeking over gains and loss-aversion are common attitudes of class 1 and 2 subjects, who make up 20.8% of the population. However, unlike class 1 subjects (8%) who display a risk-seeking attitude over losses and an ambiguity aversion attitude, class 2 subjects (12.5%) are risk-averse over losses and ambiguity-seeking. Finally, subjects in class 7 (11.3%) are risk-averse both over gains and losses. They are loss-seeking and ambiguity seeking.

As discussed in Section 2 and further highlighted below when discussing estimated updating rule parameters, investors strongly deviate from Bayes rule. In particular, they are slow in eliminating their ambiguity as new information is provided about the true return distribution. We analyzed the impact of these deviations from Bayes on the estimated preference distribution. To proceed, we estimated the preference distribution (7 classes) replacing stated probabilistic beliefs about the true return distribution with probabilistic beliefs generated using Bayes rule given the observed returns. Table 8 in the Appendix presents the estimated distribution. We find that estimated distributions imposing or not Bayes rule do not qualitatively or quantitatively differ, suggesting estimated preferences are robust to departures from Bayes rule.

Individual level estimates

We perform a more in-depth analysis of the distribution of each preference parameter by computing posterior estimates of the individual-specific parameters using Bayes theorem.[6] These are generated using

$$(\bar{\beta}_n, \bar{\tau}_n) = \sum_{c=1}^{C} h_{nc}(\Theta)(\boldsymbol{\beta}_c, \tau_c) \tag{17}$$

.

Risk aversion. We estimate risk aversion parameters in the gain and loss domains respectively. Median values of the risk aversion coefficient for gains and for losses are 0.885 (with mean=0.941 and s.d=0.393) and 0.437 (with mean= 0.708 and s.d=0.744) respectively. Non-parametric one-sided signed tests show that both medians are less than one (p-value=0.021 for gains and p-value=0.000 for losses). We also find that these medians are different using a non-parametric two-sided signed test (p-value 0.000). These results confirm the findings of diminishing marginal utility for gains (Booij and Van de Kuilen, 2009) and of sensitivity to outcomes diminishes for losses (Abdellaoui, Bleichrodt, lHaridon, and Van Dolder, 2016), which are consistent with the predictions of prospect theory (i.e risk aversion for gains and risk-seeking for losses).

We now move to the analysis at the individual level. The coefficients of risk aversion coefficients vary between 0.226 and 1.679 with an interquartile range of [0.708, 1.174] for gains and between 0.134 and 2.060 with an interquartile range of [0.156, 0.508] for losses. Concave utility is the common shape for gains (61.36%), but a substantial fraction of subjects have convex utility for gains. Convexity is clearly the most common shape for losses (76.14%). There are few subjects with concave utility for losses. In general, the most common attitude for risk is risk aversion over gains and risk-seeking over losses. Indeed, we find that 50% of subjects are risk-averse in gains and risk-seeking in losses, 12.5% are risk-seeking in gains and risk-averse in losses. Consequently, at an individual level, the reflection effect[7] in risk attitude is widely observed (62.5%) in the population.

[6] Train (2009) discusses individual level estimation and presents Monte Carlo evidence.

[7] A reflection effect occurs when the risk aversion coefficient over gains and the risk aversion coefficient over losses fall on the same side of 1

Amongst these subjects that display the reflection effect in risk attitude, a large fraction (80%) are risk-averse in gains and risk-seeking in losses, while the remaining fraction is risk-seeking in gains and risk-averse in losses. Only 11.36% of subjects have risk behavior in line with Expected Utility Theory (concave utility in both domains). Similar to our results, Booij and Van de Kuilen (2009) and Abdellaoui, Bleichrodt, and Kammoun (2013) also found the predominance pattern of a concave–convex shape of the utility function, implying diminishing sensitivity in both domains of outcomes. Moreover, Chakravarty and Roy (2009) and Baucells and Villasís (2010) have reported the reflection effect.

Loss aversion. Our estimated median value of the loss aversion coefficients is 2.148 (with mean=3.847 and s.d=5.191), which is quite consistent with the estimate of 2.25 obtained by Kahneman and Tversky (1979). The non-parametric two-sided sign test rejects the null that the median is equal to 1 (p-value=0.000). This implies that there is significant evidence of loss averse behavior in the population, i.e. the loss aversion coefficient is significantly higher than 1. Once again, we observe considerable heterogeneity at the individual level. In fact, the interquartile range is [0.288, 4.398] with a minimum of 0.005 and a maximum of 21.351. Moreover, a large majority (65.91%) of our subjects are loss averse, while a small minority of investors exhibit gain seeking behavior. These results are consistent with prior experiment and field studies reviewed in Booij, Van Praag, and Van De Kuilen (2010), Wakker (2010), and Barberis (2013).

Ambiguity aversion. The median value of the ambiguity aversion coefficient is 0.230 (with mean=-0.192 and s.d=1.253), which is significantly different from 0 (the p-value of the non-parametric two-sided signed tests is 0.000). This result suggests that at the aggregate level, the subjects of our sample are ambiguity averse. The distribution of ambiguity aversion coefficient is left-skewed, ranging from -3.640 to 0.552 with an interquartile range of [-0.035, 0.472]. The classification of the subjects according to the type of their preferences revealed that 69.32% of them are ambiguity averters. These results are consistent with prior studies conducted in the laboratory (see Trautmann and Van De Kuilen (2015)) as well as studies from the field ((Akay, Martinsson, Medhin, and Trautmann, 2012; Dimmock, Kouwenberg, and Wakker, 2015)).

Ambiguity updating. Figure 5 shows the distribution of the estimated updating rule parameters ($\delta_1; \delta_2$). Median values of δ_1 and δ_2 are 1.024 (with mean=1.187 and s.d=0.796) and 0.192 (with mean= .333 and s.d=0.418) respectively. Consistent with our finding in Chapter 1, these results suggest that subjects are conservative by placing relatively more weight on prior belief relative to recent information. We find considerable variation at the individual level, in particular for δ_1. The estimated values vary between 0.001 and 3.942 with an interquartile range of [0.899, 1.286] for δ_1 and between 0.001 and 2.659 with an interquartile range of [0.084, 0.446] for δ_2. These results are consistent with those of Bellemare et al. (2018) who analyze the same updating rule using a related experimental design.

A handful of other papers have studied individual heterogeneity in belief updating. Similar to our results, recent studies show that there is considerable heterogeneity in belief updating behavior, with people most often updating conservatively (see Ambuehl and Li (2018) for updating in a choice experiment about urn composition, Buser, Gerhards, and Van der Weele (2016) on updating about relative performance on cognitive tasks, and Mobius, Niederle, Niehaus, and Rosenblat (2011) on updating in response to positive and negative feedback about own performance).

Relationship between parameters

Table 2 presents OLS regressions of each individual level parameter on other parameters. Standard errors are clustered on the class each investor is assigned to on the basis of their posterior probabilities discussed above. We find a positive and significant relationship (1% level) between risk aversion over gains and losses (columns (1) and (2)). Moreover, ambiguity aversion is significantly and positively related to risk aversion over gains (1% level), but significantly and negatively related to risk aversion over losses (1% level, columns (2) and (4)). Loss aversion is weakly negatively related to risk aversion over losses and ambiguity aversion (10%). Finally, we find a significant positive relation (1%

level in column (5), 10% level in column (6)) between weight placed on the prior (δ_1) and weight placed on recent information (δ_2). The later suggests that investors deviating from Bayes rule by underweighting new information also tend to underweight their own prior. The combination of both forms of (under) weighting implies updating inertia, leading to persistence of ambiguity throughout the experiment. Table 6 in the Appendix expands these regressions by additionally including socio-economic information for each investor (gender, age, education level, field of study, and a measure of numeracy). Results in Table 2 are robust to the addition of these control variables, consistent with few of these additional variables having a significant effect of the preference estimates.

Optimal investment interval

We analyze for each class the optimal investment interval length given the estimated model parameters reported above. We proceed by first randomly selecting one of the possible returns distributions to serve as the true return distribution. Thirty independent draws were then generated from the selected true distribution. We simulated at the investor level the reported updating of ambiguity using parameters $(\delta_1; \delta_2)$ and the sequences of draws generated from the true return distribution using equation (14). We compute the optimal prospect value (OPV) of a given class using equation (1) at the beginning of each subsequent investment interval given class specific values of $(\theta, \theta, \lambda, \alpha)$. The OPV before the first investment interval is based on the prior probabilities presuming that all possible return distributions are equally likely. OPV is computed by evaluating (1) at the optimal investment level given a fixed investment interval length of h periods, where OPV calculations are repeated for $h = 1, 2, 3, 5 \,\text{and}\, 6$. Setting $h = 1$ implements treatment H in our experiment. Other values of h implement the decision environment in treatment L of the experiment, additionally allowing investment interval lengths of 2 and 6 periods. We summed the OPV over the number of rounds for a given h[8] to cover a total of 30 periods. This yields an aggregated optimal prospect value (AOPV) for each value h.

[8]The number of rounds is equal to 30 divided by h. It is 30, 15, 10, 6 and 5 for h equal to 1, 2, 3, 5 and 6 respectively.

We then determine the sequence specific optimal investment interval $h^* \in \{1, 2, 3, 5, 6\}$ as the value of h maximizing the AOPV. We repeated this exercise 100 times for each class, providing 100 different sequences of 30 draws from a true distribution. The optimal investment interval length is defined as $\bar{h}^*$, the average value of h^* the over the 100 different sequences of 30 draws. We compute the sampling distribution of $\bar{h}^*$ by repeating the steps described above 100 times for each class, each time using a new random draw from the estimated asymptotic distribution of the relevant model parameters for the class.

Figure 6 presents the sampling distribution of $\bar{h}^*$ for each class and for the investor population as a whole. The bottom right hand graph plots the predicted sampling distribution for all investors in our sample. The distribution is bi-modal, with a clear probability mass for short investment interval (1 period), hinting that ambiguity aversion dominates the effects of loss aversion.

This insight is better seen by looking at the predicted sampling distributions for each class separately. We find that the sampling distribution of optimal investment interval is concentrated at a interval length of 1 period for classes 1, 5, and 6. This is consistent with the preference estimates of Table 1 showing above average levels of ambiguity aversion and below levels of loss aversion for classes 1 and 5. The prediction for Class 6 is surprising given the estimated loss aversion parameter of 4.398, clearly above the average value in our population and above corresponding figures normally reported in the literature. This provides an indication that the estimated value of ambiguity aversion for this class ($\hat{\alpha} = 0.23$) is high enough to swamp the presence of significantly large loss aversion. The sampling distribution for class 7 is primarily concentrated near an optimal investment interval of l. An opposite prediction emerges for class 4 investors. Our estimates predict a long investment interval reflecting the high degree of loss aversion combined with ambiguity neutrality of this class. Predictions are mixed for classes 2 and 3, with sampling distributions covering a wide range of optimal investment intervals. Dispersions of both sampling distributions reflect the relatively lower statistical precision of estimated parameters for both classes. Finally, Figure 9 replicates the analysis for all investors, assuming the later update their ambiguity using Bayes rule. Results are very similar to those above, with a clear majority of investors preferring the most frequent portfolio evaluation frequency possible.

The simulation exercise above illustrates the importance of ambiguity aversion on investment decisions in the experiment, and highlights the need of ambiguity-averse investors to evaluate frequently their portfolio. These predictions are consistent with our data on ambiguity updating in the experiment. In particular, we have showed that learning in treatment L is slower than in treatment H both for Bayesian and non-Bayesian updaters. Ambiguity-averse investors are predicted to have a clear preference for $h = 1$, as in treatment H.

5 Conclusion

We analyzed the optimal frequency of portfolio evaluation in the presence of ambiguity using a new experimental design in which investors faced structured ambiguity at the beginning of the experiment. Stated probabilistic beliefs about the true return distribution of a stock served as a key variable to separate investment behavior under ambiguity from investment behavior under risk. This design allowed to estimate the joint distribution of preferences (risk, ambiguity, and loss aversion) and ambiguity updating rules in our sample of investors.

We found that investors update their probabilistic beliefs regarding the returns distribution of a stock slowly relative to Bayes rule. Our model estimates suggested this occurred mostly because investors placed relatively little weight on new information relative to their prior, resulting in ambiguity that persisted for a large share of investors. We also found considerable preference heterogeneity across investors in the population. Interestingly, we found that loss aversion and ambiguity aversion can co-exist at the individual level for a significant share of investors. Optimal frequency of portfolio evaluation for these investors balanced the relative strength of both preferences. We used our model estimates to predict the distribution of optimal frequency of portfolio evaluation and found that approximatively 70% of investors prefer very frequent portfolio evaluations, reflecting that ambiguity aversion dominates loss aversion.

It is tempting yet misleading to conclude that any policy that will enhance learning under ambiguity will have significant welfare benefits for a large share of investors. While our analysis suggests investors update and eliminate ambiguity slowly relative to Bayes' rule, enforcing the latter does not substantially affect the predicted distribution of optimal frequency of portfolio evaluations given the strength of ambiguity aversion measured in this paper. Upstream interventions by financial advisors aimed at limiting initial ambiguity through a smaller set of possible return distributions may have a better chance of downplaying the effects of ambiguity aversion on investment behavior.

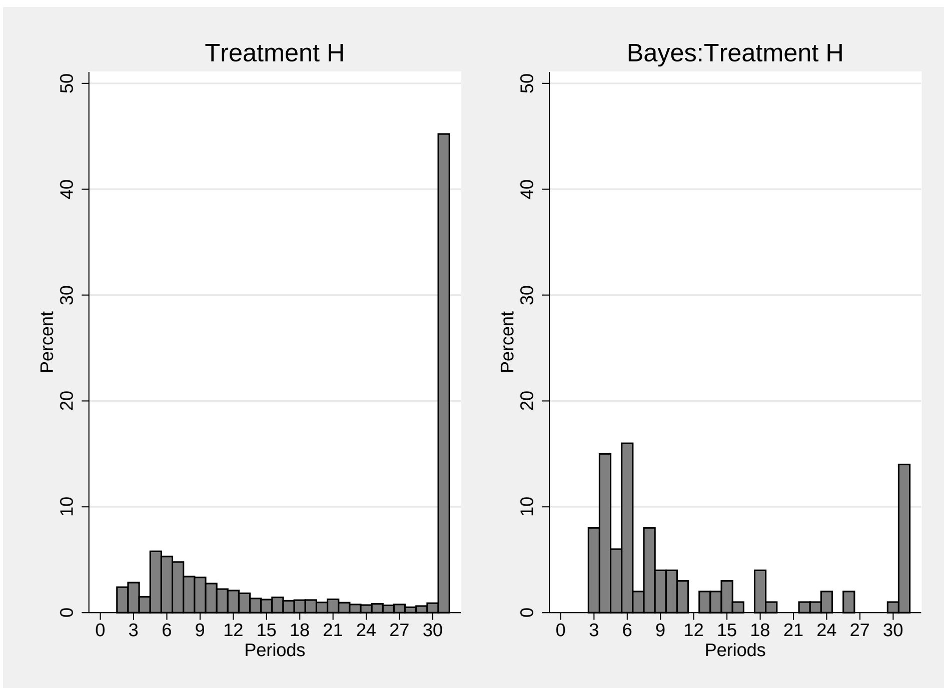

Figure 2: Left panel plots the distribution of experimental periods starting from which investors correctly assign a subjective probability greater than 90% on the true return distribution of the stock in treatment H. Right panel plots the corresponding distribution assuming investors are Bayesian.

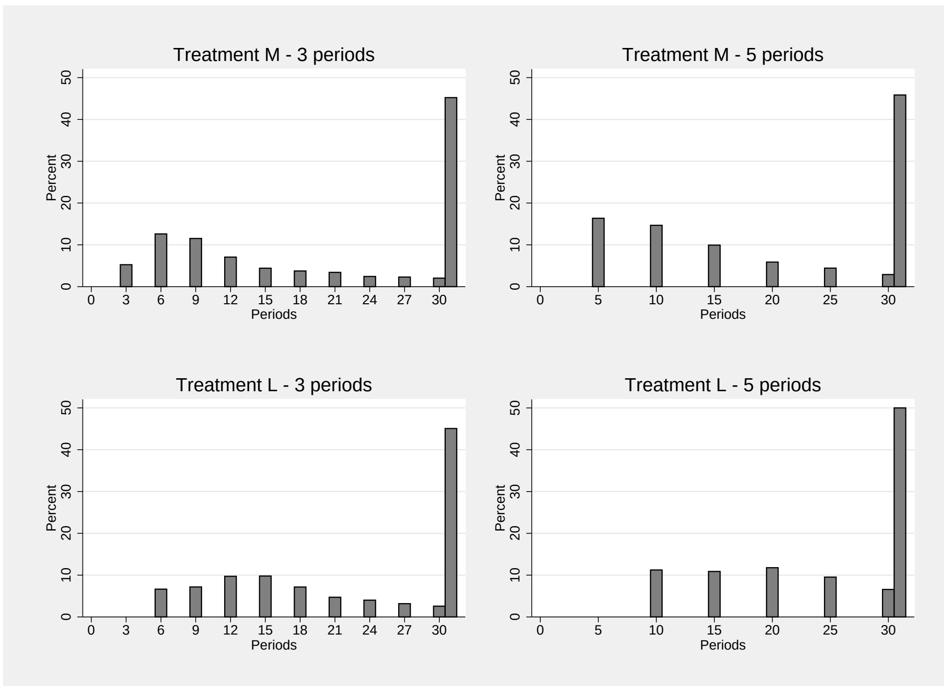

Figure 3: Distribution of experimental periods starting from which investors correctly assign a subjective probability greater than 90% on the true return distribution of the stock in treatments M and L as a function of the number of periods of the investment interval.

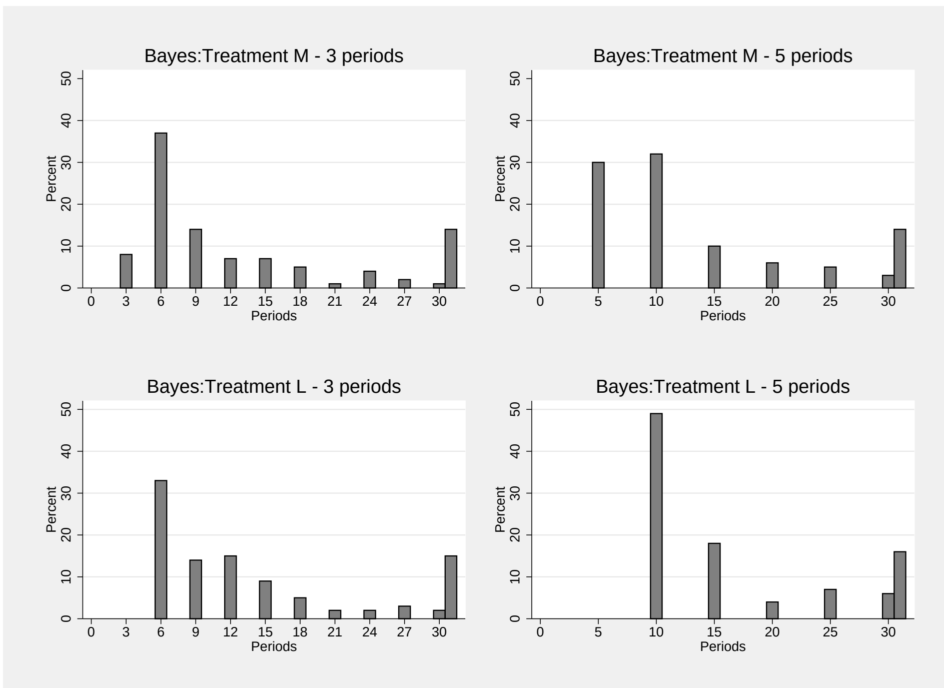

Figure 4: Distribution of experimental periods predicted using Bayes rule starting from which investors correctly assign a subjective probability greater than 90% on the true return distribution of the stock in treatments M and L as a function of the number of periods of the investment interval.

Classes	Shares	Risk aversion parameters over gains	Risk aversion parameters over losses	Loss aversion parameters	Ambiguity aversion parameters	Noise parameters
1	0.08	1.174	0.156	1.660	0.472	0.333
	(2.649)	(47.370)	(3.208)	(7.333)	(2.580)	(12.821)
2	0.125	1.679	2.060	2.148	-0.035	42.683
	(3.334)	(2.156)	(9.292)	(0.636)	(-0.491)	(0.491)
3	0.068	1.098	0.444	21.351	-0.084	5.341
	(2.477)	(1.916)	(13.366)	(0.673)	(-0.638)	(0.668)
4	0.114	1.198	0.389	6.804	0.015	0.498
	(3.167)	(113.28)	(25.675)	(18.672)	(3.140)	(27.665)
5	0.237	0.708	0.438	0.286	0.552	0.330
	(4.567)	(11.500)	(3.266)	(2.147)	(6.545)	(5.608)
6	0.263	0.885	0.134	4.398	0.230	1.203
	(4.882)	(19.008)	(6.224)	(8.824)	(4.223)	(9.199)
7	0.113	0.226	1.973	0.005	-3.640	0.079
	(3.201)	(2.590)	(10.152)	(1.130)	(-6.274)	(2.628)
Median		0,885	0,437	2,148	0,23	0,498
Mean		0,941	0,708	3,848	-0,192	6,203
Std. Deviation		0,393	0,744	5,191	1,253	13,886

Table 1: Estimated preference parameters of the latent class model with seven classes (z ratios in parentheses)

Variables	(1) θ	(2) θ'	(3) λ	(4) α	(5) δ_1	(6) δ_2
Risk av. over gains (θ)		1.356**	12.34*	2.181***	0.705	0.575
		(0.442)	(5.639)	(0.423)	(0.407)	(0.708)
Risk av. over losses (θ')	0.477***		-7.125*	-1.338***	-0.591*	-0.366
	(0.0793)		(3.399)	(0.238)	(0.267)	(0.414)
Loss aversion (λ)	0.0260	-0.0427		-0.0578	-0.0116	-0.0143
	(0.0153)	(0.0221)		(0.0354)	(0.00689)	(0.0283)
Ambiguity aversion (α)	0.337***	-0.588***	-4.231*		-0.246	-0.186
	(0.0434)	(0.103)	(2.068)		(0.153)	(0.243)
Wght on prior (δ_1)	0.0190	-0.0454	-0.149	-0.0429		0.474*
	(0.0133)	(0.0295)	(0.183)	(0.0290)		(0.227)
Wght recent inf. (δ_2)	0.0130	-0.0236	-0.153	-0.0272	0.397***	
	(0.0139)	(0.0223)	(0.384)	(0.0313)	(0.0871)	
Constant	0.533**	-0.438	-3.234	-0.997	0.960***	-0.357
	(0.149)	(0.448)	(1.950)	(0.568)	(0.176)	(0.473)
Observations	88	88	88	88	88	88
R-squared	0.766	0.815	0.365	0.851	0.249	0.226

Robust standard errors adjusted for 7 clusters in parentheses

$***p < 0.01, **p < 0.05, *p < 0.1$

Table 2: OLS regressions of determinants of individual level parameters

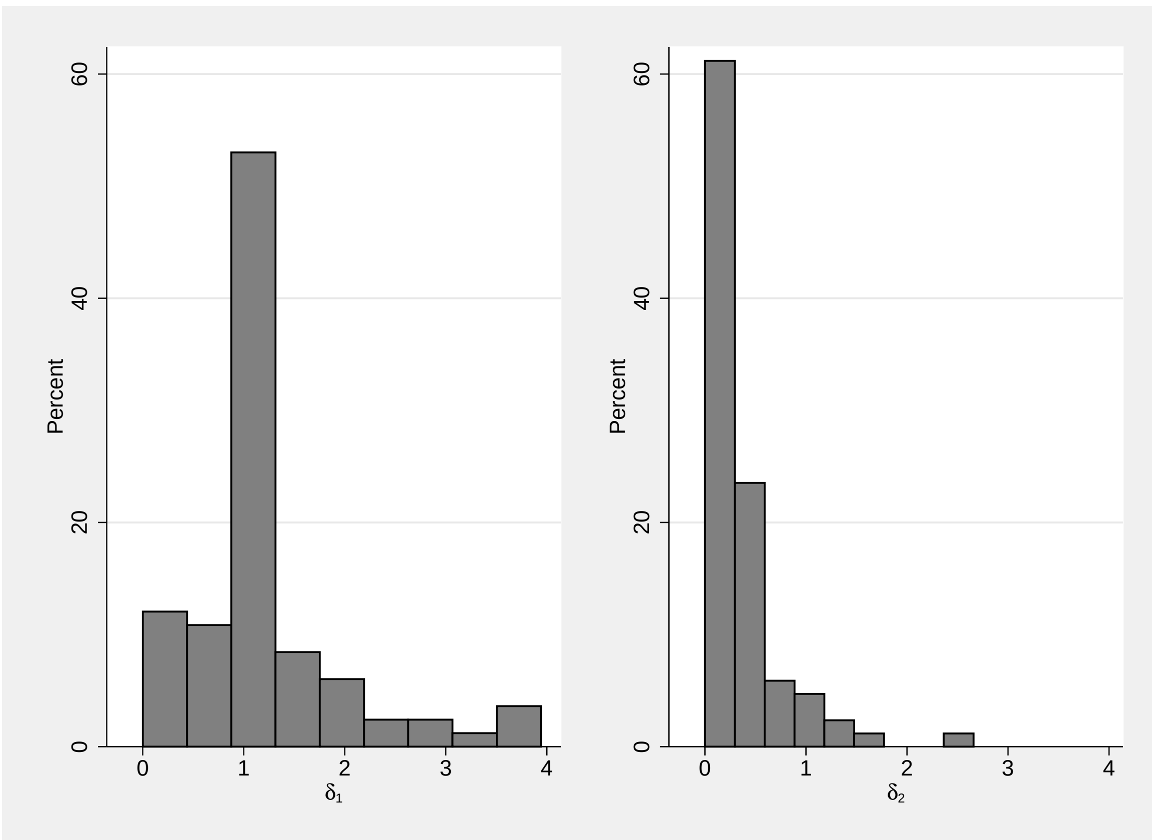

Figure 5: Distributions of estimated updating rule parameters δ_1 and δ_2

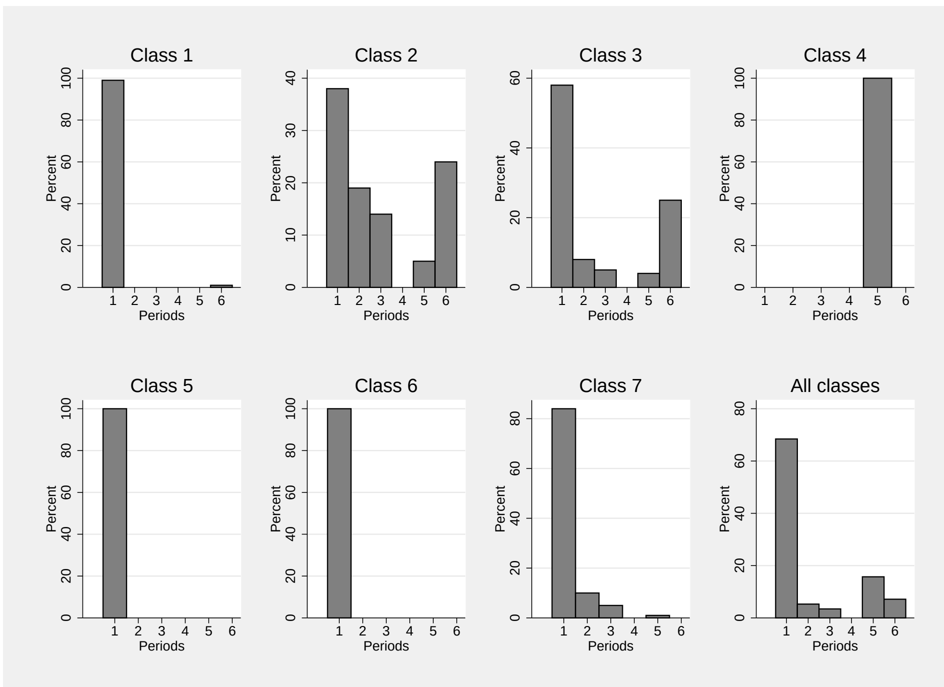

Figure 6: Sampling distributions of $\bar{h}^*$ for the seven classes and the investor population as a whole

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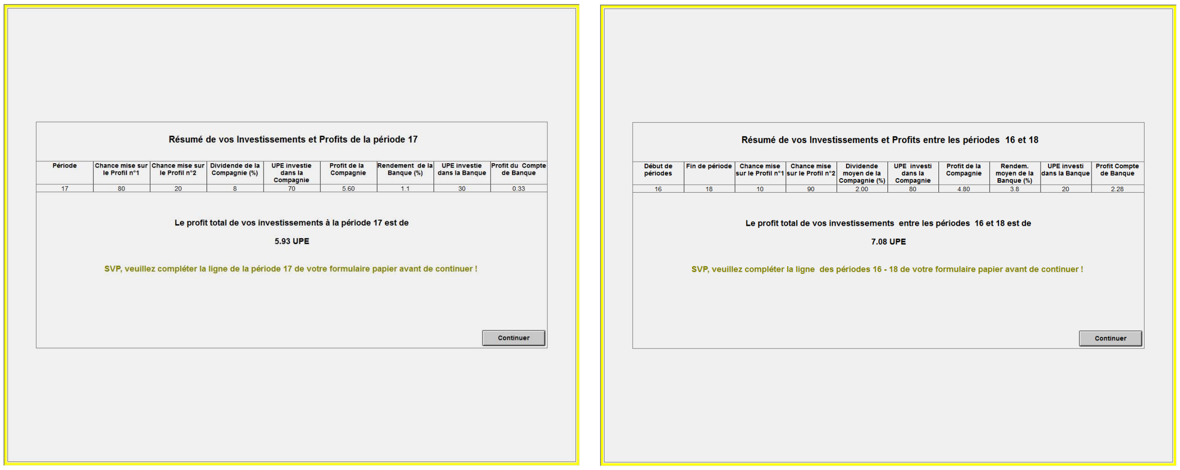

Figure 7: Screenshots of the feedback screen for treatments H and M (left) and for treatment L (right).

Tableau d'enregistrement : **Compagnie H** Numéro ID : __ __ __ __ __

Périodes	Votre évaluation de la probabilité que la *Compagnie* ait		UPE investies dans la *Compagnie*	Profit de la *Compagnie*		UPE investies dans le *Compte bancaire*	Gain du *Compte bancaire*	Profit total
	le Profil n° 1	le Profil n° 2		Gains (+)	Pertes (-)			
1								
2								
…								
30								
							Profits totaux	

Tableau d'enregistrement : **Compagnie M** Numéro ID : __ __ __ __ __

Périodes	Votre évaluation de la probabilité que la *Compagnie* ait		UPE investies dans la *Compagnie*	Profit de la *Compagnie*		UPE investies dans le *Compte bancaire*	Gain du *Compte bancaire*	Profit total
	le Profil n° 1	le Profil n° 2		Gains (+)	Pertes (-)			
1								
2								
…								
30								
							Profits totaux	

Tableau d'enregistrement : **Compagnie L** Numéro ID : __ __ __ __ __

Périodes	Votre évaluation de la probabilité que la *Compagnie* ait		UPE investies dans la *Compagnie*	Profit de la *Compagnie*		UPE investies dans le *Compte bancaire*	Gain du *Compte bancaire*	Profit total
	le Profil n° 1	le Profil n° 2		Gains (+)	Pertes (-)			
1-5								
6-10								
11-15								
16-20								
21-25								
26-30								
							Profits totaux	

Tableau d'enregistrement : **Compagnie H** Numéro ID : __ __ __ __ __

Périodes	Votre évaluation de la probabilité que la *Compagnie* ait		UPE investies dans la *Compagnie*	Profit de la *Compagnie*		UPE investies dans le *Compte bancaire*	Gain du *Compte bancaire*	Profit total
	le Profil n° 1	le Profil n° 2		Gains (+)	Pertes (-)			
1								
2								
…								
30								
							Profits totaux	

Tableau d'enregistrement : **Compagnie M** Numéro ID : __ __ __ __ __

Périodes	Votre évaluation de la probabilité que la *Compagnie* ait		UPE investies dans la *Compagnie*	Profit de la *Compagnie*		UPE investies dans le *Compte bancaire*	Gain du *Compte bancaire*	Profit total
	le Profil n° 1	le Profil n° 2		Gains (+)	Pertes (-)			
1								
2								
…								
30								
							Profits totaux	

Tableau d'enregistrement : **Compagnie L** Numéro ID : __ __ __ __ __

Périodes	Votre évaluation de la probabilité que la *Compagnie* ait		UPE investies dans la *Compagnie*	Profit de la *Compagnie*		UPE investies dans le *Compte bancaire*	Gain du *Compte bancaire*	Profit total
	le Profil n° 1	le Profil n° 2		Gains (+)	Pertes (-)			
1-3								
4-6								
7-9								
10-12								
13-15								
16-18								
19-21								
22-24								
25-27								
28-30								
							Profits totaux	

Figure 8: Record sheet with examples of recording tables for all three treatments. Example for low flexibility/frequency with five periods (left hand) and with three periods (right hand).

Treatment sequence	Investment interval		Total
	3 periods	5 periods	
HML	4	3	7
LMH	3	5	8
MLH	6	4	10
Total	13	12	25

Table 3: Frequency of investors dropped as a function of the treatment sequence and the length of the investment interval in treatments L and M.

Table 4: Latent Class Models with different numbers of classes

Classes	**Log-Likelihood**	**Parameters**	**AIC**	**BIC**
1	-17519	5	35048	35060
2	-17147	11	34316	34343
3	-16480	17	32994	33036
4	-16333	23	32712	32769
5	-16197	29	32452	32524
6	-16094	35	32258	32345
7	-16014	41	32110	32212

Subjects	Posterior probability of Subject s belonging class C_i							Maxi-	Max	Max	Max
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	mum	$> .9$	$> .95$	$> .99$
1	1	0	0	0	0	0	0	1	1	1	1
2	0	0	0	0	0	1	0	1	1	1	1
3	0	0	0	0	0.999	0.001	0	0.999	1	1	1
4	0	0	0	1	0	0	0	1	1	1	1
5	0	0	0	0	1	0	0	1	1	1	1
6	0	0	0	0	0	0	1	1	1	1	1
7	0	0	0	0	0.003	0	0.997	0.997	1	1	1
8	0	0	0	1	0	0	0	1	1	1	1
9	0	0	0	0	0	0	1	1	1	1	1
10	0	0	0	0	0	1	0	1	1	1	1
11	1	0	0	0	0	0	0	1	1	1	1
12	0	0.960	0.040	0	0	0	0	0.960	1	1	0
13	0	0	0	0	0.123	0.874	0.003	0.874	0	0	0
14	0	1	0	0	0	0	0	1	1	1	1
15	0	0	0	0	0.594	0.406	0	0.594	0	0	0
16	0	0	0	0	0.998	0.002	0	0.998	1	1	1
17	0	0	0	1	0	0	0	1	1	1	1
18	0	1	0	0	0	0	0	1	1	1	1
19	0	0	0	0	1	0	0	1	1	1	1
20	0	0	0	0	0.994	0.006	0	0.994	1	1	1
21	0	0	0	0	0.920	0.079	0.001	0.920	1	0	0
22	0	0	0	0	0	0	1	1	1	1	1
23	0	0	0	0	0	1	0	1	1	1	1
24	0	0	0	1	0	0	0	1	1	1	1
25	0	0	0	0	0	1	0	1	1	1	1
26	0	0	0	0	0.008	0.982	0.011	0.982	1	1	0
27	0	0	0	0	0	1	0	1	1	1	1
28	0	0	0	0	0.141	0.859	0	0.859	0	0	0
29	0	0	0	0	0.001	0.001	0.998	0.998	1	1	1
30	0	0	1	0	0	0	0	1	1	1	1
31	0	0	1	0	0	0	0	1	1	1	1
32	0	0	0	0	0.999	0.001	0	0.999	1	1	1
33	0	0	0	0	1	0	0	1	1	1	1
34	0	0	0	0	0	1	0	1	1	1	1
35	0	0.079	0.921	0	0	0	0	0.921	1	0	0
36	0	0	0	0	0.994	0.006	0	0.994	1	1	1
37	0	0	0	0	0	1	0	1	1	1	1
38	0	1	0	0	0	0	0	1	1	1	1
39	0.004	0	0	0.996	0	0	0	0.996	1	1	1
40	0	0	1	0	0	0	0	1	1	1	1
41	0	0	0	0	0	1	0	1	1	1	1
42	0	0	0	0	0	1	0	1	1	1	1
43	0.002	0	0	0	0.998	0	0	0.998	1	1	1
44	0	0	0	1	0	0	0	1	1	1	1
45	1	0	0	0	0	0	0	1	1	1	1
46	0	0	0	0	0.999	0.001	0	0.999	1	1	1
47	0	0	0	0	1	0	0	1	1	1	1
48	0	1	0	0	0	0	0	1	1	1	1
49	0	0.999	0.001	0	0	0	0	0.999	1	1	1
50	0	0	0	0	0.998	0.002	0	0.998	1	1	1
51	0	0	0	0	0	1	0	1	1	1	1
52	0	0	0	0	0	0	1	1	1	1	1
53	0	0	0	0	0	1	0	1	1	1	1
54	0.998	0	0	0	0.002	0	0	0.998	1	1	1
55	0	0	0	1	0	0	0	1	1	1	1
56	0	0	0	0	0	0	1	1	1	1	1
57	0	0	0	0	0	1	0	1	1	1	1
58	0	0	0	0	0.001	0.999	0	0.999	1	1	1
59	0	0	0	0	1	0	0	1	1	1	1
60	0	0	0	0	0	1	0	1	1	1	1
61	0	0	0	0	1	0	0	1	1	1	1
62	0	1	0	0	0	0	0	1	1	1	1
63	0	1	0	0	0	0	0	1	1	1	1
64	1	0	0	0	0	0	0	1	1	1	1
65	0	0	0	0	0	1	0	1	1	1	1
66	0	0	0	0	0	0.985	0.015	0.985	1	1	0
67	0	0	1	0	0	0	0	1	1	1	1
68	0	0	0	0	1	0	0	1	1	1	1
69	0	0	0	0	0.071	0.927	0.002	0.927	1	0	0
70	0	0	0	0	0	1	0	1	1	1	1
71	0	1	0	0	0	0	0	1	1	1	1
72	0	0	0	0	0	0	1	1	1	1	1
73	1	0	0	0	0	0	0	1	1	1	1
74	0	0	1	0	0	0	0	1	1	1	1
75	0	0	0	0	1	0	0	1	1	1	1
76	0	0	0	0	0.998	0	0.002	0.998	1	1	1
77	0	0	0	0	0	0	1	1	1	1	1
78	0	0	0	0	1	0	0	1	1	1	1
79	1	0	0	0	0	0	0	1	1	1	1
80	0	0	0	1	0	0	0	1	1	1	1
81	0	0	0	1	0	0	0	1	1	1	1
82	0	0	0	0	0	1	0	1	1	1	1
83	0	0	0	0	0.002	0.059	0.939	0.939	1	0	0
84	0	0	0	0	1	0	0	1	1	1	1
85	0	1	0	0	0	0	0	1	1	1	1
86	0	1	0	0	0	0	0	1	1	1	1
87	0	0	0	1	0	0	0	1	1	1	1
88	0	0	0	0	0	1	0	1	1	1	1
Average								**0.988**	**0.966**	**0.920**	**0.886**

Table 5: Class specific posterior probabilities for each investor ($N = 88$).

Variables	(1) θ	(2) θ'	(3) λ	(4) α	(5) δ_1	(6) δ_2
Risk av. over gains (θ)		1.389**	12.60*	2.206***	0.570	0.500
		(0.414)	(5.630)	(0.382)	(0.394)	(0.711)
Risk av. over losses (θ')	0.473***		-6.980*	-1.318***	-0.542*	-0.355
	(0.0759)		(3.320)	(0.217)	(0.249)	(0.449)
Loss aversion (λ)	0.0251	-0.0408*		-0.0547	-0.0216	0.00212
	(0.0137)	(0.0201)		(0.0309)	(0.0114)	(0.0325)
Ambiguity aversion (α)	0.341***	-0.598***	-4.250*		-0.116	-0.247
	(0.0395)	(0.0965)	(2.081)		(0.134)	(0.272)
Wght on prior (δ_1)	0.0161	-0.0450	-0.307	-0.0213		0.542*
	(0.0146)	(0.0336)	(0.314)	(0.0276)		(0.259)
Wght recent inf. (δ_2)	0.0108	-0.0225	0.0230	-0.0344	0.414***	
	(0.0140)	(0.0249)	(0.343)	(0.0326)	(0.0575)	
Female	-0.0429	0.0728	0.771	0.0665	0.169	-0.512
	(0.0230)	(0.0449)	(0.852)	(0.0540)	(0.147)	(0.266)
Group=5	-0.00834	0.0573	0.0880	0.0525	0.431	-0.150
	(0.0143)	(0.0350)	(0.210)	(0.0384)	(0.262)	(0.149)
Age	-0.00711	0.00131	0.0320	-0.000466	0.0704	-0.0138
	(0.0107)	(0.0175)	(0.136)	(0.0277)	(0.0939)	(0.145)
Age^2	4.01e-05	1.45e-05	-0.000276	0.000104	-0.000933	0.000175
	(0.000141)	(0.000218)	(0.00180)	(0.000367)	(0.00109)	(0.00164)
Numericy	0.0157	-0.0214	0.354	-0.0515	0.232**	-0.188
	(0.0156)	(0.0247)	(0.261)	(0.0398)	(0.0840)	(0.185)
Bachelor	-0.129**	0.185**	2.833	0.267**	0.454	-0.0878
	(0.0500)	(0.0543)	(2.851)	(0.0979)	(0.407)	(0.390)
Master and more	-0.0616	0.115	1.288	0.207	-0.234	0.358
	(0.0531)	(0.0845)	(2.038)	(0.109)	(0.239)	(0.350)
Science	-0.0790	0.0795	1.462	0.254	-0.482**	-0.00315
	(0.0431)	(0.0550)	(1.468)	(0.132)	(0.190)	(0.0810)
Economics and Finance	-0.0493*	0.139*	-0.737	0.202**	-0.0322	0.242
	(0.0238)	(0.0572)	(0.851)	(0.0729)	(0.155)	(0.282)
Constant	0.804**	-0.709	-7.040	-1.383	-0.869	0.360
	(0.221)	(0.567)	(4.984)	(0.739)	(2.054)	(3.152)
Observations	88	88	88	88	88	88
R-squared	0.802	0.838	0.431	0.874	0.392	0.312

Robust standard errors adjusted for 7 clusters in parentheses
$***p < 0.01, **p < 0.05, *p < 0.1$

Table 6: OLS regressions of determinants of individual level parameters

A Estimation results with Bayes rule updating assumption

A.1 Information criteria

Table 7: Latent Class Models with different numbers of classes

Classes	Log-Likelihood	Parameters	AIC	BIC
1	-17555	5	35120	35132
4	-16400	23	32846	32903
6	-16185	35	32440	32527
7	-16110	41	32302	32404

A.2 Estimated parameters

Table 8: Estimate latent class model with seven classes (z ratios in parentheses)

Classes	Shares	Risk aversion parameters over gains	Risk aversion parameters over losses	Loss aversion parameters	Ambiguity aversion parameters	Noise parameters
1	0.08	1.549	0.153	4.396	0.248	0.709
	(2.646)	(44.081)	(6.575)	(10.572)	(12.187)	(11.367)
2	0.126	1.585	2.025	1.802	-0.043	32.111
	(3.331)	(2.273)	(8.907)	(0.762)	(-0.565)	(0.559)
3	0.068	1.080	0.449	19.925	-0.093	4.969
	(2.476)	(1.907)	(13.357)	(0.683)	(-0.651)	(0.675)
4	0.102	1.245	0.340	8.552	0.022	0.659
	(3.000)	(84.991)	(22.120)	(16.910)	(2.0123)	(22.028)
5	0.242	0.916	0.086	5.709	0.302	1.443
	(4.568)	(17.752)	(4.5225)	(7.593)	(3.975)	(7.816)
6	0.114	0.218	1.985	0.004	-4.095	0.083
	(3.229)	(2.048)	(8.3925)	(0.858)	(-3.670)	(2.085)
7	0.268	0.560	0.247	0.479	0.589	0.232
	(4.915)	(17.981)	(6.695)	(4.598)	(6.949)	(9.837)
			summary statistics			
Median		0.916	0.247	2.346	0.254	0.709
Mean		0.920	0.647	4.308	-0,226	4.929
Standard Deviation		0.436	0.769	5.014	1.401	10.377

A.3 Correlation matrix

Table 9: Spearman pairwise correlation matrix of preference parameters

	$\boldsymbol{\theta}$	$\boldsymbol{\theta'}$	$\boldsymbol{\lambda}$
Risk aversion over gains ($\boldsymbol{\theta}$)			
Risk aversion over losses ($\boldsymbol{\theta'}$)	0.088		
Loss aversion ($\boldsymbol{\lambda}$)	0.653*	-0.317*	
Ambiguity aversion ($\boldsymbol{\alpha}$)	-0.295*	-0.611*	-0.151

*Correlation coefficients significant at the 5% level or lower

A.4 Optimal evaluation frequency

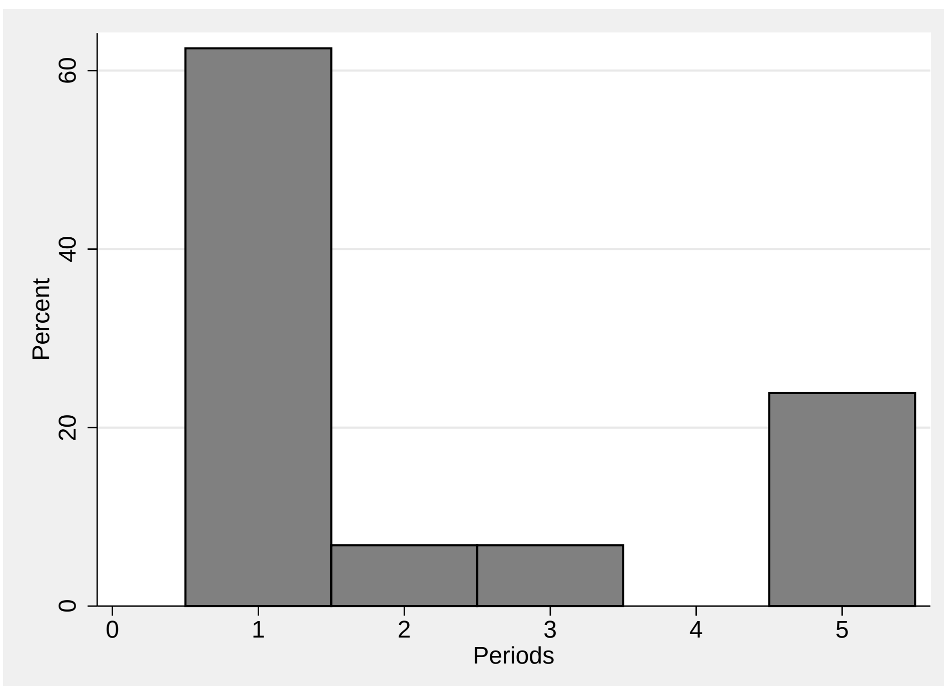

Figure 9: Distribution of optimal portfolio evaluation frequency in the population of investors assuming updating using Bayes rule.